Scuola Matematica Interuniversitaria, Perugia 2019

FUNCTIONAL ANALYSIS

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Description: This in an introductory course in Functional Analysis: it covers the main results on Banach and Hilbert spaces as well as some applications. The theoretical concepts will be illustrated with many examples that cover spaces of sequences and functions. To a large extent, the course will be self contained and full notes will be provided to students; we shall follow [1; Chapters 1-3, 5-6, 9] and [2; Chapters 2-5].

The program of the course is listed below.

1. Banach spaces

Normed spaces and complete norms; examples
Quotient spaces
Continuous linear operators and functionals
Hahn-Banach theorem and its geometrical forms
Strong and weak convergence
Weak* topology and Banach-Alaoglu theorem
Duals of normed spaces
Reflexive spaces
The Uniform Bounded Principle
Open Mapping theorem
Closed Graph theorem
Adjoint operators
Compact operators on Banach spaces and their spectrum

2. Hilbert Spaces

Projections and orthogonality
Orthonormal bases
Riesz representation theorem
Lax-Milgram theorem
Spectral decomposition of self-adjoint compact operators

3. Applications

Sobolev spaces and Poincaré inequality
Laplace operator and weak solutions
The spectrum of the Laplace operator

Prerequisites: A good course in Real Analysis and some familiarity with *topological vector spaces* [2, Sections 1.1, 1.2 and 1.3], *Lebesgue measure and L^p spaces* [1, Sections 4.1, 4.2].

References

[1] Haïm Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer 2011.

[2] Walter Rudin, Functional Analysis, 2nd ed., McGraw-Hill, New York, 1991.