Scuola Matematica Interuniversitaria, Perugia, 2019

Course: Numerical Analysis

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Description of the course

The course will consist of an introduction to Network Science, including mathematical foundations and computational aspects. This emerging field represents an ideal venue for the introduction of a variety of tools from linear algebra, discrete mathematics (graph theory), mathematical modelling (differential equations on networks, Markov processes) and their numerical treatment using state-of-the art computational methods. The students taking the course will learn basic Numerical Analysis by seeing it ``in action" in a fascinating and important application field rather than in isolation.

Topics to be covered:

1) Introduction to Network Science: motivation, historical remarks

2) Basic concepts: graphs, adjacency matrices, graph Laplacians, connectivity, spectral properties, digraphs

3) Walks on graphs (deterministic and random)

4) Centrality and communicability measures via eigenvectors and via matrix functions (resolvent, exponential, …)

5) Iterative solution of large, sparse linear systems and eigenvalue problems arising in Network Science

6) Computation of matrix functions: diagonalization-based and Krylov subspace-based methods

Exercises and laboratory: The students will be expected to solve illustrative problems both by hand (including rigorous mathematical proofs) and by computer.

Prerequisites: a good grasp of basic Linear Algebra and Mathematical Analysis. Some knowledge of Matlab or similar software environment is highly desirable but not absolutely necessary

Suggested readings:

Ernesto Estrada and Philip K. Night, ``A First Course in Network Theory", Oxford University Press, 2015.

Nicholas J. Higham, ``Functions of Matrices. Theory and Computation", Society for Industrial and Applied Mathematics, 2008.