

DIFFERENTIAL GEOMETRY
SMI, Perugia 2019
LIVIU ORNEA

Prerequisites

Linear algebra: vector spaces, scalar products, norms, symmetric endomorphisms, eigenvalues, multilinear forms.
Differential and integral calculus of one and several real variables.
Elementary theory of ODE's.

Syllabus

1. Differentiable manifolds.

- 1.1 Definitions, examples.
- 1.2 Tangent space, differentiable maps.
- 1.3 Immersions, submersions, submanifolds. Regular value theorem.
- 1.4 Lie groups.
- 1.5 Vector fields, bracket. Flows. Lie derivative.
- 1.6 Distributions. Frobenius theorem.

2. Tensors and differential forms.

- 2.1 Tensor algebra, exterior algebra.
- 2.2 Tensor bundles. Sections.
- 2.3 Exterior differential. Lie derivative of forms.
- 2.4 De Rham cohomology. Poincaré lemma.
- 2.5 Integration on manifolds.

3. Vector bundles

- 3.1 Definitions. Constructions.
- 3.2 Connections in vector bundles. Parallel transport.

4. Riemannian manifolds

- 3.1 Definitions. Examples. Riemannian coverings.
- 3.2 Levi Civita connection.
- 3.3 Riemannian curvature. Sectional curvature.
- 3.4 Geodesics.

Recommended texts:

[1] F. Warner, Foundations of differentiable manifolds and Lie groups. Corrected reprint of the 1971 edition. Graduate Texts in Mathematics, 94. Springer-Verlag, New York-Berlin, 1983. ix+272 pp.

[2] W. Boothby, An introduction to differentiable manifolds and Riemannian geometry. Second edition. Pure and Applied Mathematics, 120. Academic Press, Inc., Orlando, FL, 1986. xvi+430 pp.

- NB 1)** The prerequisites (especially ODE's) should be regarded as mere wishful thinking.
- 2)** The syllabus will be adapted according to the participants' background and preferences.
- 3)** I can lecture in English or Italian, according to the participants' preferences.