ITERATIVE METHODS FOR LARGE-SCALE SADDLE POINT PROBLEMS

TENTATIVE PROGRAM (M. BENZI)

(Cortona, 9–20 May, 2022)

Lecture schedule:

Lecture 1: Presentation of the course, introduction to saddle point problems, examples, main assumptions, solvability conditions, algebraic and spectral properties of saddle-point matrices.

Lecture 2: Overview of solution algorithms, direct vs. iterative methods, Schur complement reduction, null space methods, augmented Lagrangian formulation, factorization of saddle-point matrices, remarks on sparse direct solvers (fill-in, elimination graph, reorderings, minimum degree, band-reducing heuristics).

Lecture 3: Stationary iterations (Arrow-Hurwicz, Uzawa, inexact and preconditioned variants), Hestenes' Method of Multipliers, introduction to Krylov subspace methods.

Lecture 4: More on Krylov methods: convergence analysis; flexible, inexact, and preconditioned variants, asymptotic convergence rates for sequences of problems of increasing size, field-of-values analysis.

Lecture 5: Block preconditioners for saddle point problems (block diagonal and block triangular, constraint preconditioning), spectral analysis, exact vs. inexact, augmented Lagrangian-based.

Lecture 6: Block diagonal/triangular preconditioners for the Stokes and Navier-Stokes equations; Hermitian-skew Hermitian (HSS) and Modified HSS (MHSS) preconditioning.

Lecture 7: Dimensional Splitting and Relaxed Dimensional Factorization preconditioning for the Stokes and Navier-Stokes equations.

Lecture 8: Double/multiple saddle point problems, the coupled Darcy-Stokes problem.

Lecture 9: Approximate/incomplete factorization methods, multilevel techniques.

Lecture 10: Loose ends, conclusions, discussion and perspectives.

Course prerequisites: excellent knowledge of linear algebra; previous exposure to numerical analysis including numerical linear algebra, optimization, and numerical methods for PDEs.

Selected references:

M. Benzi, G. H. Golub, and J. Liesen, *Numerical Solution of Saddle Point Problems*, Acta Numerica 2005, pp. 1–137.

D. Boffi, F. Brezzi and M. Fortin, *Mixed Finite Element Methods and Applications*, Springer, 2013.

H. Elman, D. Silvester and A. Wathen, *Finite Elements and Fast Iterative Solvers, Second Edition*, Oxford Science Publication, 2014.

M. Rozloznik, *Saddle Point Problems and their Iterative Solution*, Birkhäuser, Cham, 2018.

Y. Saad, *Iterative Methods for Sparse Linear Systems, Second Edition*, SIAM, Philadelphia, 2003.