Splines and PDEs: a basic introduction to Isogeometric Analysis

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Presentation

Spline functions are ubiquitous, although sometimes hidden, in numerical methods. They are probably the most popular approximation tool in several contexts. Besides their theoretical interest, they have applications in several branches of the sciences. In particular, they are the core of Isogeometric Analysis (IgA), a quite recent and well-established paradigm for the numerical treatment of Partial Differential Equations, which aims to build an integrated framework for Modeling and Analysis. The success of IgA mainly relies on the B-spline basis: a special and renowned basis to represent splines that enjoys several important properties from both theoretical and computational point of view. The course aims to present an introduction to IgA with a special focus on the advantages stemming from the properties of B-splines.

Program

Introduction

1. Basics on elliptic problems: weak formulation
2. Basics on Galerkin methods

Basics on splines and B-splines

1. Polynomial splines
2. B-splines: definition and main properties
3. B-splines as a basis of polynomial splines

B-splines: geometric properties and approximation properties

1. B-splines in geometric modeling
2. Knot insertion and refinements
3. Basics on approximation power of spline spaces

Towards the multivariate setting

1. Tensor-product splines and B-splines

Some case studies

1. Basics on Isogeometric discretizations
2. Onedimensional diffusion-advection-reaction problems
3. Basics on elliptic problems on a bivariate domain

References

C. de Boor: *A Practical Guide to Splines*, Springer, 2001.

J.A. Cottrell, T.J.R. Hughes, Y. Bazilevs: *Isogeometric Analysis: Toward Integration of CAD and FEA*, John Wiley & Sons, 2009.

T. Lyche, C. Manni, H. Speleers: *Foundations of spline theory: B-splines, spline approximation, and hierarchical refinement*. In: T. Lyche, C. Manni, and H. Speleers (eds.) Splines and PDEs: From Approximation Theory to Numerical Linear Algebra, Lecture Notes in Mathematics 2219, Springer International Publishing, pp. 1–76, 2018**.**

C. Manni, H. Speleers: *Standard and non-standard CAGD tools for isogeometric analysis: A tutorial*. In: A. Buffa and G. Sangalli (eds.) IsoGeometric Analysis: A New Paradigm in the Numerical Approximation of PDEs, Lecture Notes in Mathematics 2161, Springer International Publishing, pp. 1–69, 2016.

Prerequisites

Contents of the basic courses of Analysis, Geometry and Algebra of the undergraduate degree. Knowledge of basic notions of Numerical Analysis. Knowledge of very basic notions of Functional Analysis.